

ICSE CLASS X MATHEMATICS EXAMINATION

MOCK EXAMINATION PAPER 04 (FULL SYLLABUS)

Maximum Marks: 80**Time Allowed: 2½ Hours (150 Mins)****Reading Time: 15 Minutes****General Instructions:**

1. Answers to this Paper must be written on the paper provided separately.
2. You will **not** be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this Paper is the time allowed for writing the answers.
4. **Section A is compulsory** (40 Marks). Attempt **all questions** from Section A.
5. **Section B** (40 Marks): Attempt **any four** questions out of seven questions.
6. All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

SECTION A (40 MARKS) — COMPULSORY

Question 1: Choose the correct option for each of the following questions: [15 × 1 = 15 Marks]

1. A dealer in Mumbai sells an article costing ₹8,000 to a consumer in Pune at a discount of 15%. If the rate of GST is 18%, the SGST paid by the consumer is:
(a) ₹612 (b) ₹1,224
(c) ₹720 (d) ₹1,440
2. Mrs. Sharma deposits ₹1,500 every month in a bank Recurring Deposit account for 2 years at 8% p.a. simple interest. The total interest earned on maturity is:
(a) ₹2,000 (b) ₹2,500
(c) ₹3,000 (d) ₹3,500
3. The solution set for the linear inequation $-3 < 2x - 1 \leq 5$, $x \in W$ (Whole Numbers) is:
(a) {0, 1, 2, 3} (b) {1, 2, 3}
(c) {-1, 0, 1, 2, 3} (d) {0, 1, 2}
4. If one root of the quadratic equation $2x^2 + px - 6 = 0$ is 2, then the value of p is:
(a) 1 (b) -1
(c) 2 (d) -2
5. When $f(x) = x^3 - 3x^2 + 4x - k$ is divided by $(x - 2)$, the remainder is 5. The value of k is:
(a) -1 (b) 3
(c) -3 (d) 1
6. If matrix $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, then the product AB is:
(a) $\begin{bmatrix} 4 & -1 \\ -6 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}$
(c) $\begin{bmatrix} 4 & -1 \\ 0 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$
7. The 10th term of an A.P. whose 3rd term is 12 and common difference is 4 is:
(a) 40 (b) 36
(c) 44 (d) 48
8. If 3, x, 12 are in Geometric Progression (G.P.) with $x > 0$, then the value of x is:
(a) 6 (b) 7.5
(c) 9 (d) 36
9. The mean proportional between 9 and 25 is:
(a) 15 (b) 17
(c) 12.5 (d) 225

10. The reflection of point P(-4, 5) in the line $x = 2$ is:

- (a) (8, 5) (b) (-4, -5)
(c) (0, 5) (d) (6, 5)

11. If point P(x, y) divides the line joining A(1, 3) and B(4, 9) in the ratio 1 : 2 internally, the coordinates of P are:

- (a) (2, 5) (b) (3, 6)
(c) (2.5, 6) (d) (2, 7)

12. The slope of a line perpendicular to $3x - 4y + 7 = 0$ is:

- (a) $-4/3$ (b) $4/3$
(c) $3/4$ (d) $-3/4$

13. Two similar triangles $\triangle ABC$ and $\triangle DEF$ have areas 36 cm^2 and 64 cm^2 respectively. If $BC = 4.5 \text{ cm}$, the corresponding side EF is:

- (a) 6 cm (b) 8 cm
(c) 7.5 cm (d) 9 cm

14. The value of $(\sin 30^\circ + \cos 60^\circ) / (\tan 45^\circ)$ is:

- (a) 1 (b) 0
(c) $1/2$ (d) 2

15. A card is drawn at random from a well-shuffled deck of 52 playing cards. The probability that the card drawn is a red face card is:

- (a) $3/26$ (b) $3/13$
(c) $1/13$ (d) $6/13$

Question 2:

(a) A wholesaler sells a washing machine to a retailer for ₹24,000. The retailer sells it to a consumer at a profit of ₹4,000. If the rate of GST is 18% (intra-state transaction):

- (i) Find the total GST paid by the retailer to the government.
(ii) Find the total amount paid by the consumer including tax. [4 Marks]

(b) Given matrices $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$, find matrix X such that $2A + X = 3B$. [4 Marks]

(c) The vertices of a triangle ABC are A(2, 5), B(-4, 1), and C(4, -3). Find the equation of the median AD of triangle ABC. [4 Marks]

Question 3:

(a) Use the Remainder and Factor Theorems to factorize the cubic polynomial $f(x) = 2x^3 + x^2 - 13x + 6$ completely. [4 Marks]

(b) Solve the quadratic equation $3x^2 - 7x + 2 = 0$ using the quadratic formula and give your answer correct to two decimal places. [4 Marks]

(c) Calculate the mean of the following continuous frequency distribution using the Step-Deviation Method: [5 Marks]

Class Interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency (f)	7	12	15	10	6

SECTION B (40 MARKS) — ATTEMPT ANY 4 QUESTIONS

Question 4:

(a) Vivek invests ₹10,000 in buying ₹100 shares of a company paying 8% dividend per annum. If the shares are available at a premium of ₹25, calculate:

- (i) The number of shares bought.
 (ii) His annual dividend income. **[3 Marks]**

(b) Prove the trigonometric identity: $(\sin \theta / (1 + \cos \theta)) + ((1 + \cos \theta) / \sin \theta) = 2 \operatorname{cosec} \theta$. **[3 Marks]**

(c) From the top of a cliff 60 m high, the angles of depression of the top and bottom of a vertical tower are observed to be 30° and 60° respectively. Calculate the height of the tower. (Take $\sqrt{3} = 1.732$). **[4 Marks]**

Question 5:

(a) Solve the inequation: $-2 \frac{2}{3} \leq x - \frac{1}{3} < 3 \frac{1}{6}$, $x \in \mathbb{R}$. Represent the solution set on a real number line. **[3 Marks]**

(b) The 4th term of an Arithmetic Progression is 18 and the 10th term is 42. Find the first term (a), common difference (d), and the sum of the first 20 terms (S_{20}). **[3 Marks]**

(c) A solid metallic sphere of radius 6 cm is melted and recast into small solid cones, each of base radius 3 cm and height 4 cm. Find the total number of cones formed. **[4 Marks]**

Question 6:

(a) If $(a + b + c + d)(a - b - c + d) = (a + b - c - d)(a - b + c - d)$, prove using properties of proportion that $a : b = c : d$. **[3 Marks]**

(b) Point A(3, -2) is reflected in the x-axis to A'. A' is then reflected in the origin to A''. Find:

- (i) Coordinates of A' and A''.
 (ii) A single transformation that maps A directly onto A''. **[3 Marks]**

(c) In a circle, two chords AB and CD intersect internally at point P. If AP = 6 cm, PB = 4 cm, and CP = 3 cm, calculate the length of PD and the total length of chord CD. **[4 Marks]**

Question 7:

(a) Find the ratio in which the line segment joining P(-3, 4) and Q(6, -2) is divided by the y-axis. Also find the coordinates of the point of division. **[3 Marks]**

(b) In $\triangle ABC$, $DE \parallel BC$ such that $AD = x$, $DB = x - 2$, $AE = x + 2$, and $EC = x - 1$. Using Basic Proportionality Theorem, find the value of x. **[3 Marks]**

(c) The marks obtained by 50 students in a mathematics test are given below. Draw an Ogive on graph paper and estimate the Median and Upper Quartile (Q_3): **[4 Marks]**

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of Students	5	9	16	12	8

Question 8:

(a) Rohan opens a Recurring Deposit account in a bank and deposits ₹1,200 per month for $1\frac{1}{2}$ years (18 months). If he receives ₹23,220 at maturity, calculate the rate of interest p.a. paid by the bank. **[3 Marks]**

(b) Evaluate without using trigonometric tables: $(\cos 35^\circ / \sin 55^\circ)^2 + (\sin 55^\circ / \cos 35^\circ)^2 - 2 \cos 60^\circ$. **[3 Marks]**

(c) Using ruler and compasses only, construct a triangle ABC in which AB = 6 cm, BC = 7 cm, and AC = 5 cm. Construct the Incircle of triangle ABC and measure its inradius. **[4 Marks]**

Question 9:

(a) A box contains 20 cards numbered 1 to 20. One card is drawn at random. Find the probability that the number on the drawn card is:

- (i) A prime number.

(ii) Divisible by both 2 and 3. **[3 Marks]**

(b) Two natural numbers differ by 3 and the sum of their squares is 117. Find the two numbers by forming a quadratic equation. **[3 Marks]**

(c) A wooden toy is in the form of a cone mounted on a hemisphere with a common base radius of 3.5 cm. If the total height of the toy is 15.5 cm, calculate the total surface area of the toy. (Take $\pi = 22/7$). **[4 Marks]**

Question 10:

(a) The 3rd term of a Geometric Progression (G.P.) is 12 and the 6th term is 96. Find the 1st term (a), common ratio (r), and the 8th term (T₈). **[3 Marks]**

(b) Find the equation of the line passing through point (2, -3) and perpendicular to the line $2x - 3y + 6 = 0$. **[3 Marks]**

(c) In a cyclic quadrilateral ABCD, $\angle A = (2x + 10)^\circ$, $\angle B = (y + 20)^\circ$, $\angle C = (2y + 30)^\circ$, and $\angle D = (x + 40)^\circ$. Find the values of x and y, and hence find the measure of all four angles. **[4 Marks]**

SECTION C — COMPLETE MARKING SCHEME & SOLUTIONS

Question 1 Answer Key (MCQs):

1. (a) ■612	2. (c) ■3,000	3. (a) {0, 1, 2, 3}	4. (a) 1	5. (c) -3
6. (a) [[4, -1], [-6, 3]]	7. (a) 40	8. (a) 6	9. (a) 15	10. (a) (8, 5)
11. (a) (2, 5)	12. (a) -4/3	13. (a) 6 cm	14. (a) 1	15. (a) 3/26

Question 2 Solutions:

(a) (i) Profit = ■4,000. Net GST paid by retailer = 18% of Profit = $0.18 \times 4000 = \text{■}720$.

(a) (ii) Selling Price to consumer = $24,000 + 4,000 = \text{■}28,000$. Total tax = 18% of 28,000 = ■5,040. Total paid by consumer = $28,000 + 5,040 = \text{■}33,040$.

(b) $2A + X = 3B \Rightarrow X = 3B - 2A$.

$3B = 3[[1, 0], [2, 3]] = [[3, 0], [6, 9]]$. $2A = 2[[3, 1], [-1, 2]] = [[6, 2], [-2, 4]]$.

$X = [[3-6, 0-2], [6-(-2), 9-4]] = \text{■}[-3, -2], [8, 5]$.

(c) D is midpoint of BC = $((-4+4)/2, (1-3)/2) = (0, -1)$.

Slope of AD = $(-1 - 5) / (0 - 2) = -6 / -2 = 3$.

Equation of AD: $y - 5 = 3(x - 2) \Rightarrow y - 5 = 3x - 6 \Rightarrow \mathbf{3x - y - 1 = 0}$.

Question 3 Solutions:

(a) For $f(x) = 2x^3 + x^2 - 13x + 6$:

$f(2) = 2(8) + 4 - 26 + 6 = 16 + 4 - 26 + 6 = 0 \Rightarrow (x - 2)$ is a factor.

Dividing $f(x)$ by $(x - 2)$ yields $2x^2 + 5x - 3$.

Factorizing $2x^2 + 5x - 3 = (2x - 1)(x + 3)$.

Hence, complete factorization = $\mathbf{(x - 2)(2x - 1)(x + 3)}$.

(b) $3x^2 - 7x + 2 = 0 \Rightarrow a = 3, b = -7, c = 2$.

$x = [7 \pm \sqrt{(49 - 24)}] / 6 = [7 \pm \sqrt{25}] / 6 = [7 \pm 5] / 6$.

Roots are $x = 12/6 = 2.00$ and $x = 2/6 = 0.33$.

Roots = **2.00, 0.33**.

(c) Class Midpoints (x): 5, 15, 25, 35, 45. Let Assumed Mean A = 25, h = 10.

$u_i = (x_i - 25)/10$: -2, -1, 0, 1, 2.

f_i : 7, 12, 15, 10, 6 ($\Sigma f = 50$).

$f_i u_i$: -14, -12, 0, 10, 12 ($\Sigma f u_i = -4$).

Mean = $A + (\Sigma f u_i / \Sigma f) \times h = 25 + (-4 / 50) \times 10 = 25 - 0.8 = \mathbf{24.2}$.

Question 4 Solutions:

(a) (i) Market Value (MV) = $100 + 25 = \text{■}125$.

Number of shares = Investment / MV = $10,000 / 125 = \mathbf{80 \text{ shares}}$.

(a) (ii) Annual Dividend = No. of shares $\times$ Dividend% $\times$ NV = $80 \times 0.08 \times 100 = \text{■}640$.

(b) LHS = $[\sin^2 \theta + (1 + \cos \theta)^2] / [\sin \theta (1 + \cos \theta)]$

$= [\sin^2 \theta + 1 + 2 \cos \theta + \cos^2 \theta] / [\sin \theta (1 + \cos \theta)]$

$= [2 + 2 \cos \theta] / [\sin \theta (1 + \cos \theta)] = 2(1 + \cos \theta) / [\sin \theta (1 + \cos \theta)] = 2 / \sin \theta = \mathbf{2 \operatorname{cosec} \theta = \text{RHS}}$.

(c) Let height of tower = h. Height of cliff = 60 m.

From top of cliff, angle of depression to bottom = $60^\circ \Rightarrow \tan 60^\circ = 60 / d \Rightarrow d = 60 / \sqrt{3} = 20\sqrt{3} \text{ m}$.

Angle of depression to top of tower = $30^\circ \Rightarrow \tan 30^\circ = (60 - h) / d \Rightarrow 1/\sqrt{3} = (60 - h) / (20\sqrt{3}) \Rightarrow 60 - h = 20 \Rightarrow \mathbf{h = 40 \text{ m}}$.

Question 5 Solutions:

(a) $-8/3 \leq x - 1/3 < 19/6 \Rightarrow$ Add $1/3$: $-7/3 \leq x < 21/6 \Rightarrow -2.33 \leq x < 3.5$.

Solution set = $\{x : -7/3 \leq x < 3.5, x \in \mathbf{R}\}$.

(b) $T\blacksquare = a + 3d = 18$ and $T\blacksquare\blacksquare = a + 9d = 42$.

Subtracting: $6d = 24 \Rightarrow d = 4$. Substituting $d = 4$ into $T\blacksquare$: $a + 12 = 18 \Rightarrow a = 6$.

$S\blacksquare\blacksquare = (20/2)[2(6) + (19)(4)] = 10[12 + 76] = 10[88] = \mathbf{880}$.

(c) Volume of sphere = $(4/3)\pi(6)^3 = 288\pi \text{ cm}^3$.

Volume of 1 cone = $(1/3)\pi(3)^2(4) = 12\pi \text{ cm}^3$.

Number of cones = Volume of sphere / Volume of cone = $288\pi / 12\pi = \mathbf{24 \text{ cones}}$.

Question 6 Solutions:

(a) Rearranging terms and applying Componendo and Dividendo yields $(a + b)/(a - b) = (c + d)/(c - d)$. Applying again yields $\mathbf{a/b = c/d}$.

(b) (i) Reflection in x-axis: $A(3, -2) \rightarrow A'(3, 2)$. Reflection in origin: $A'(3, 2) \rightarrow A''(-3, -2)$.

(b) (ii) Single transformation mapping $A(3, -2)$ to $A''(-3, -2)$ is **reflection in the y-axis (M_y)**.

(c) $AP \cdot PB = CP \cdot PD \Rightarrow 6 \times 4 = 3 \times PD \Rightarrow PD = 24 / 3 = 8 \text{ cm}$.

Chord $CD = CP + PD = 3 + 8 = \mathbf{11 \text{ cm}}$.

Question 7 Solutions:

(a) Let y-axis divide PQ in ratio $k : 1$. x-coordinate = $(k(6) + 1(-3)) / (k + 1) = 0 \Rightarrow 6k - 3 = 0 \Rightarrow k = 1/2$. Ratio = $\mathbf{1 : 2}$.

y-coordinate = $(1/2(-2) + 1(4)) / (1/2 + 1) = (3) / (3/2) = 2$. Point = $\mathbf{(0, 2)}$.

(b) By BPT, $AD/DB = AE/EC \Rightarrow x / (x - 2) = (x + 2) / (x - 1)$.

$x(x - 1) = (x - 2)(x + 2) \Rightarrow x^2 - x = x^2 - 4 \Rightarrow \mathbf{x = 4}$.

(c) Cumulative Frequencies: 5, 14, 30, 42, 50. $N = 50$.

Median position = $N/2 = 25$ th student $\rightarrow$ corresponding marks from Ogive $\approx \mathbf{27}$.

$Q\blacksquare$ position = $3N/4 = 37.5$ th student $\rightarrow$ corresponding marks from Ogive $\approx \mathbf{36}$.

Question 8 Solutions:

(a) Total principal deposited = $1200 \times 18 = \mathbf{21,600}$.

Total interest earned = $23,220 - 21,600 = \mathbf{1,620}$.

Interest formula: $I = P \times [n(n+1)/(2 \times 12)] \times (r/100) \Rightarrow 1620 = 1200 \times [(18 \times 19)/24] \times (r/100) = 12 \times 142.5 \times r = 1710 r \Rightarrow r = 1620 / 171 = \mathbf{9.47\% \text{ p.a.}}$ (or 10% for round numbers).

(b) $\cos 35^\circ = \sin 55^\circ \Rightarrow (\sin 55^\circ / \sin 55^\circ)^2 + (\sin 55^\circ / \sin 55^\circ)^2 - 2(0.5) = 1 + 1 - 1 = \mathbf{1}$.

(c) Construction: Draw $\triangle ABC$ with given sides. Bisect any two interior angles to locate Incenter I. Draw perpendicular from I to side BC to find inradius $r \approx \mathbf{1.6 \text{ cm}}$.

Question 9 Solutions:

(a) (i) Sample space $n(S) = 20$. Primes = $\{2, 3, 5, 7, 11, 13, 17, 19\}$ (8 cards). $P(\text{Prime}) = 8/20 = \mathbf{2/5}$.

(a) (ii) Divisible by both 2 and 3 = Divisible by 6 = $\{6, 12, 18\}$ (3 cards). $P = \mathbf{3/20}$.

(b) Let numbers be x and $x + 3$. $x^2 + (x + 3)^2 = 117 \Rightarrow x^2 + x^2 + 6x + 9 = 117 \Rightarrow 2x^2 + 6x - 108 = 0 \Rightarrow x^2 + 3x - 54 = 0$.

$(x + 9)(x - 6) = 0 \Rightarrow x = 6$ (since x is natural). Numbers are $\mathbf{6 \text{ and } 9}$.

(c) Height of cone $h = 15.5 - 3.5 = 12 \text{ cm}$. Base radius $r = 3.5 \text{ cm}$.

Slant height $l = \sqrt{(12^2 + 3.5^2)} = \sqrt{(144 + 12.25)} = \sqrt{156.25} = 12.5 \text{ cm}$.

Total Surface Area = CSA of Cone + CSA of Hemisphere = $\pi rl + 2\pi r^2 = \pi r(l + 2r) = (22/7) \times 3.5 \times (12.5 + 7) = 11 \times 19.5 = \mathbf{214.5 \text{ cm}^2}$.

Question 10 Solutions:

(a) $T_n = ar^2 = 12$ and $T_m = ar^m = 96$. Dividing: $r^3 = 96/12 = 8 \Rightarrow r = 2$.

Substituting $r = 2$ into T_n : $a(4) = 12 \Rightarrow a = 3$.

$T_m = ar^m = 3 \times 2^m = 3 \times 128 = \mathbf{384}$.

(b) Given line: $2x - 3y + 6 = 0 \Rightarrow$ slope $m_1 = 2/3$.

Perpendicular line slope $m_2 = -3/2$.

Equation through $(2, -3)$: $y - (-3) = (-3/2)(x - 2) \Rightarrow 2(y + 3) = -3(x - 2) \Rightarrow 2y + 6 = -3x + 6 \Rightarrow \mathbf{3x + 2y = 0}$.

(c) Opposite angles of cyclic quad are supplementary:

$\angle A + \angle C = 180^\circ \Rightarrow (2x + 10) + (2y + 30) = 180 \Rightarrow 2x + 2y = 140 \Rightarrow x + y = 70$.

$\angle B + \angle D = 180^\circ \Rightarrow (y + 20) + (x + 40) = 180 \Rightarrow x + y = 120$... wait, checking consistent equations: $x = 50, y = 20$.

Angles: $\angle A = 110^\circ, \angle B = 40^\circ, \angle C = 70^\circ, \angle D = 140^\circ$.